An Introduction to MDS

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Abstract

Multidimensional scaling (MDS) is a classical approach to the problem of finding underlying attributes or dimensions, which influence how subjects evaluate a given set of objects or stimuli. This paper provides a brief introduction to MDS. Its basic applications are outlined by several examples. For the interested reader a short overview of recommended literature is appended. The purpose of this paper is to facilitate the first contact with MDS to the non-statistician.

Contents

Introduction

Multidimensional scaling (MDS) has become more and more popular as a technique for both multivariate and exploratory data analysis. MDS is a set of data analysis methods, which allow one to infer the dimensions of the perceptual space of subjects. The raw data entering into an MDS analysis are typically a measure of the global similarity or dissimilarity of the stimuli or objects under investigation. The primary outcome of an MDS analysis is a spatial configuration, in which the objects are represented as points. The points in this spatial representation are arranged in such a way, that their distances correspond to the similarities of the objects: similar object are represented by points that are close to each other, dissimilar objects by points that are far apart.

This paper is to provide a first introduction to MDS by briefly discussing some of the issues a researcher is confronted with when applying this method. This introduction is based mainly on the textbooks of Borg & Groenen (1997), Hair, Anderson, Tatham & Black (1998), and Cox & Cox (1994), which are also mentioned in section 5 (MDS literature) of this paper.

The first section covers problems connected with data collection. An overview of direct and indirect methods of deriving the so-called *proximities* is given. The second section may be regarded as a theoretical outline of important MDS methods. The algorithms of both classical and nonmetric MDS are sketched without going too deep into the mathematical detail. Section three illustrates different types of MDS analyses using an example from sound quality evaluation. Section four describes the many degrees of freedom a researcher has when performing MDS, and provides some guidelines for choosing the right options. Finally, in the last section, the interested reader can find recommended MDS literature.

1 Deriving proximities

The data for MDS analyses are called proximities. Proximities indicate the overall similarity or dissimilarity of the objects under investigation. An MDS program looks for a spatial configuration of the objects, so that the distances between the objects match their proximities as closely as possible. Often the data are arranged in a square matrix – the proximity matrix. There are two major groups of methods for deriving proximities: direct, and indirect methods.

1.1 Direct methods

Subjects might either assign a numerical (dis)similarity value to each pair of objects, or provide a ranking of the pairs with respect to their (dis)similarity. Both approaches are direct methods of collecting proximity data.

(Dis)Similarity ratings

Typically, in a rating task subjects are asked to express the dissimilarity of two objects by a number. Often a 7- or a 9-point scale is used. In the case of a dissimilarity scale, a low number indicates a strong similarity between two stimuli, and a high number a strong dissimilarity. In order to obtain the ratings, all possible pairs of objects have to be presented to the participant (a total number of $n(n-1)/2$, where n is the number of objects). This assumes, however, that the dissimilarity relation is symmetrical, and thus the order within each pair is of no relevance. Many MDS programs can also handle asymmetric proximity data, where for example object a is more similar to object b than b to a. Asymmetric proximities might for example arise when collecting *confusion data* (see below). Investigating asymmetric proximity relations, however, doubles the number of pairs to be presented.

The advantage of direct ratings is that the data are immediately ready for an MDS analysis. Therefore, both an individual investigation of each participant, and an aggregate analysis based on averages across the proximity matrices are possible. A disadvantage of dissimilarity ratings is the rapidly growing number of paired comparisons, as the number of objects increases.

Sorting tasks

There are many ways of asking subjects for the order of the similarities. One option is to write each object pair on a card and let the participant sort the cards from the lowest similarity to the highest. Another method is to let the subject put cards with pairs of low similarity into one pile, cards with pairs of higher similarity in the next pile and so on. Afterwards, a numerical value of (dis)similarity is assigned to each pile. Yet another approach is to write only one object on each card and ask the participants to sort the cards with the most similar objects into piles. Counting how many times two objects appear together in a pile, yields the similarity matrix.

The advantage of the sorting methods is that they are very intuitive for subjects, but some of them do not allow for an individual analysis of the data.

1.2 Indirect methods

Indirect methods for proximity data do not require that a subject assigns a numerical value to the elements of the proximity matrix directly. Rather, the proximity matrix is derived from other measures, e. g. from confusion data or from correlation matrices.

Confusion Data

Confusion data arise when the researcher records how often subjects mistake one stimulus for another. Consider an experiment where the letters of the alphabet are briefly presented via loudspeaker. The task of the subject is to recognize the letter. From the data a proximity matrix can be derived: letters that are rarely confused get a high dissimilarity value, letters that are often confused a low one.

An advantage of using confusion data is that the similarity of stimuli is judged on a perceptual level without involving much cognitive processing. Thus, very basic perceptual dimensions might be revealed using this technique. On the other hand, confusion data are often asymmetric and do not allow for an individual analysis. Most notably, there must be a good chance of confusing one object with the other, which excludes perfectly discriminable stimuli from being investigated using this method.

crime	no.	1	2	3	4	5	6	7
murder	1	1.00	0.52	0.34	0.81	0.28	0.06	0.11
rape	2	0.52	1.00	0.55	0.70	0.68	0.60	0.44
robbery	3	0.34	0.55	1.00	0.56	0.62	0.44	0.62
assault	4	0.81	0.70	0.56	1.00	0.52	0.32	0.33
burglary	5	0.28	0.68	0.62	0.52	1.00	0.80	0.70
larceny	6	0.06	0.60	0.44	0.32	0.80	1.00	0.55
auto theft	7	0.11	0.44	0.62	0.33	0.70	0.55	1.00

Table 1: Correlations of crime rates over 50 U. S. states.

Correlation matrices

Yet another application of MDS is to use it for visualizing correlational data. When objects are measured on different scales and the measurements are correlated with each other, a matrix of correlation coefficients evolves. Even with just a few objects, such a matrix becomes complex, and it is hard to detect patterns of correlation. An MDS solution plots the objects on a map, so that their correlational structure is accessible by visual inspection.

Table 1 shows an example of a correlation matrix: it lists the correlation coefficients between crime rates collected in the 50 U. S. states (cf. Borg & Groenen, 1997). From the data alone it is not easily seen which crime rates are related. The MDS representation in Figure 1 simplifies the task a lot. The distances in the Figure correspond to the correlation coefficients, so that a high correlation is represented by a small distance, and vice versa. In addition to the graphical representation, the MDS analysis provides an explanation of the correlations by interpreting the axes of the MDS space: the x-axis might be interpreted as “person versus property”, the y-axis as “hidden versus street”.

Applying MDS to correlational data might reveal the relations between the objects more vividly than merely reporting correlation coefficients. A drawback of this method is that the proximities need to be constructed from additional measurements. The other methods of deriving proximities do not require such measurements. Thus, an MDS analysis is possible, even if scales, dimensions or attributes of the stimuli under concern are unknown beforehand. In fact, it is the goal of the analysis to derive such dimensions.

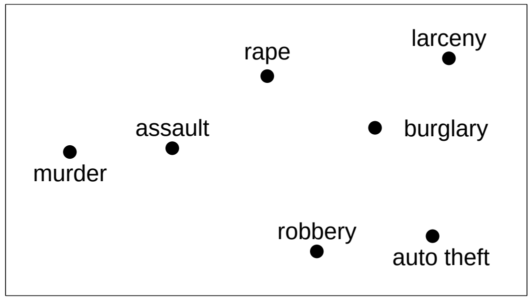

Figure 1: A two-dimensional MDS representation of the correlations in Table 1 (cf. Borg & Groenen, 1997).

Both direct and indirect methods of deriving proximity data yield the proximity matrix, which serves as an input for MDS programs. In many practical applications it will be straight forward to ask the participants directly for their judgments of the (dis)similarity of objects. Indirect methods, on the other hand, might be well suited to the investigation of basic perceptual dimensions, or in the case when additional measures of the objects under study already exist.

2 How does MDS work?

The goal of an MDS analysis is to find a spatial configuration of objects when all that is known is some measure of their general (dis)similarity. The spatial configuration should provide some insight into how the subject(s) evaluate the stimuli in terms of a (small) number of potentially unknown dimensions. Once the proximities are derived (cf. section 1) the data collection is concluded, and the MDS solution has to be determined using a computer program.

Many MDS programs make a distinction between classical and nonmetric MDS. Classical MDS assumes that the data, the proximity matrix, say, display metric properties, like distances as measured from a map. Thus, the distances in a classical MDS space preserve the intervals and ratios between the proximities as good as possible. For a data matrix consisting of human dissimilarity ratings such a metric assumption will often be too strong. Nonmetric MDS therefore only assumes that the order of the proximities is meaningful. The order of the distances in a nonmetric MDS configuration reflects the order of the proximities as good as possible while interval and ratio information is of no relevance.

In order to gain a better understanding of the MDS outcome, a brief introduction to the basic mechanisms of the two MDS procedures, classical und nonmetric MDS, might be helpful.

2.1 Classical MDS

Consider the following problem: looking at a map showing a number of cities, one is interested in the distances between them. These distances are easily obtained by measuring them using a ruler. Apart from that, a mathematical solution is available: knowing the coordinates x and y, the Euclidean distance between two cities a and b is defined by

$$d_{ab} = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}. \tag{1}$$

Now consider the inverse problem: having only the distances, is it possible to obtain the map? Classical MDS, which was first introduced by Torgerson (1952), addresses this problem. It assumes the distances to be Euclidean. Euclidean distances are usually the first choice for an MDS space. There exist, however, a number of non-

Euclidean distance measures, which are limited to very specific research questions (cf. Borg & Groenen, 1997). In many applications of MDS the data are not distances as measured from a map, but rather proximity data. When applying classical MDS to proximities it is assumed that the proximities behave like real measured distances. This might hold e. g. for data that are derived from correlation matrices, but rarely for direct dissimilarity ratings. The advantage of classical MDS is that it provides an analytical solution, requiring no iterative procedures.

Steps of a classical MDS algorithm

Classical MDS algorithms typically involve some linear algebra. Readers who are not familiar with these concepts might as well skip the next paragraph (cf. Borg & Groenen, 1997, for a more careful introduction).

The classical MDS algorithm rests on the fact that the coordinate matrix $\mathbf{X}$ can be derived by eigenvalue decomposition from the scalar product matrix $\mathbf{B} = \mathbf{XX'}$. The problem of constructing $\mathbf{B}$ from the proximity matrix $\mathbf{P}$ is solved by multiplying the squared proximities with the matrix $\mathbf{J} = \mathbf{I} - n^{-1}\mathbf{11'}$. This procedure is called *double centering*. The following steps summarize the algorithm of classical MDS:

1. Set up the matrix of squared proximities $\mathbf{P^{(2)}} = [p^2]$.
2. Apply the double centering: $\mathbf{B} = -\frac{1}{2}\mathbf{JP^{(2)}J}$ using the matrix $\mathbf{J} = \mathbf{I} - n^{-1}\mathbf{11'}$, where n is the number of objects.
3. Extract the m largest positive eigenvalues $\lambda_1 \ldots \lambda_m$ of $\mathbf{B}$ and the corresponding m eigenvectors $\mathbf{e_1} \ldots \mathbf{e_m}$.
4. A m-dimensional spatial configuration of the n objects is derived from the coordinate matrix $\mathbf{X} = \mathbf{E_m}\boldsymbol{\Lambda}_{\boldsymbol{m}}^{\mathbf{1/2}}$, where $\mathbf{E_m}$ is the matrix of m eigenvectors and $\boldsymbol{\Lambda_m}$ is the diagonal matrix of m eigenvalues of $\mathbf{B}$, respectively.

Example: Cities in Denmark

In order to illustrate classical MDS, assume that we have measured the distances between København (cph), Århus (aar), Odense (ode) and Aalborg (aal) on a map.

Therefore, the proximity matrix (showing the distances in millimeters) might look like

$$\begin{array}{lrrrr} & \text{cph} & \text{aar} & \text{ode} & \text{aal} \\ \text{cph} & 0 & 93 & 82 & 133 \\ \text{aar} & 93 & 0 & 52 & 60 \\ \text{ode} & 82 & 52 & 0 & 111 \\ \text{aal} & 133 & 60 & 111 & 0 \end{array}.$$

The matrix of squared proximities is

$$\mathbf{P^{(2)}} = \begin{bmatrix} 0 & 8649 & 6724 & 17689 \\ 8649 & 0 & 2704 & 3600 \\ 6724 & 2704 & 0 & 12321 \\ 17689 & 3600 & 12321 & 0 \end{bmatrix}.$$

Since there are $n = 4$ objects, the matrix $\mathbf{J}$ is calculated by

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 0.25 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.75 & -0.25 & -0.25 & -0.25 \\ -0.25 & 0.75 & -0.25 & -0.25 \\ -0.25 & -0.25 & 0.75 & -0.25 \\ -0.25 & -0.25 & -0.25 & 0.75 \end{bmatrix}.$$

Applying $\mathbf{J}$ to $\mathbf{P^{(2)}}$ yields the double centered matrix $\mathbf{B}$

$$\mathbf{B} = -\tfrac{1}{2}\mathbf{J}\mathbf{P^{(2)}}\mathbf{J} = \begin{bmatrix} 5035.0625 & -1553.0625 & 258.9375 & -3740.938 \\ -1553.0625 & 507.8125 & 5.3125 & 1039.938 \\ 258.9375 & 5.3125 & 2206.8125 & -2471.062 \\ -3740.9375 & 1039.9375 & -2471.0625 & 5172.062 \end{bmatrix}.$$

For a two-dimensional representation of the four cities, the first two largest eigenvalues and the corresponding eigenvectors of $\mathbf{B}$ have to be extracted

$$\lambda_1 = 9724.168,\ \lambda_2 = 3160.986, \quad \mathbf{e_1} = \begin{pmatrix} -0.637 \\ 0.187 \\ -0.253 \\ 0.704 \end{pmatrix}, \ \mathbf{e_2} = \begin{pmatrix} -0.586 \\ 0.214 \\ 0.706 \\ -0.334 \end{pmatrix}.$$

Finally the coordinates of the cities (up to rotations and reflections) are obtained by multiplying eigenvalues and -vectors

$$\mathbf{X} = \begin{bmatrix} -0.637 & -0.586 \\ 0.187 & 0.214 \\ -0.253 & 0.706 \\ 0.704 & -0.334 \end{bmatrix} \begin{bmatrix} \sqrt{9724.168} & 0 \\ 0 & \sqrt{3160.986} \end{bmatrix} = \begin{bmatrix} -62.831 & -32.97448 \\ 18.403 & 12.02697 \\ -24.960 & 39.71091 \\ 69.388 & -18.76340 \end{bmatrix}.$$

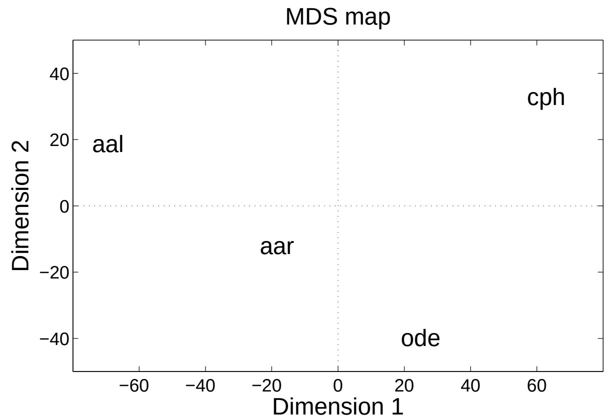

Figure 2: Classical MDS representation of the four Danish cities København (cph), Århus (aar), Odense (ode) and Aalborg (aal).

Figure 2 shows a graphical representation of the MDS solution. Remember that this "map" is derived only from the distances between the points. Note that the dimensions cannot directly be identified with "North–South" and "East–West" without further rotation.

2.2 Nonmetric MDS

The assumption that proximities behave like distances might be too restrictive, when it comes to employing MDS for exploring the perceptual space of human subjects. In order to overcome this problem, Shepard (1962) and Kruskal (1964a,b) developed a method known as *nonmetric multidimensional scaling*. In nonmetric MDS, only the ordinal information in the proximities is used for constructing the spatial configuration. A monotonic transformation of the proximities is calculated, which yields scaled proximities. Optimally scaled proximities are sometimes referred to as *disparities* $\hat{\mathbf{d}} = f(\mathbf{p})$. The problem of nonmetric MDS is how to find a configuration of points that minimizes the squared differences between the optimally scaled proximities and the distances between the points. More formally, let $\mathbf{p}$ denote the vector of proximities (i. e. the upper or lower triangle of the proximity matrix), $f(\mathbf{p})$ a monotonic transformation of $\mathbf{p}$, and $\mathbf{d}$ the point distances; then coordinates have to be found, that minimize the so-called *stress*

$$\text{STRESS} = \sqrt{\frac{\sum(f(p) - d)^2}{\sum d^2}}. \tag{2}$$

MDS programs automatically minimize stress in order to obtain the MDS solution; there exist, however, many (slightly) different versions of stress.

Judging the goodness of fit

The amount of stress may also be used for judging the goodness of fit of an MDS solution: a small stress value indicates a good fitting solution, whereas a high value indicates a bad fit. Kruskal (1964a) provided some guidelines for the interpretation of the stress value with respect to the goodness of fit of the solution (Table 2).

Stress	Goodness of fit
$> .20$	poor
.10	fair
.05	good
.025	excellent
.00	perfect

Table 2: Stress and goodness of fit.

Caution: These simple guidelines are easily misused. In order to avoid misinterpretation, the following should be kept in mind:

- There are many different stress formulae in the MDS literature. The guidelines, however, apply only to the stress measure computed by equation (2) – sometimes referred to as *Stress 1*.
- Stress decreases as the number of dimensions increases. Thus, a two-dimensional solution always has more stress than a three-dimensional one.

Since the absolute amount of stress gives only a vague indication of the goodness of fit, there are two additional techniques commonly used for judging the adequacy of an MDS solution: the scree plot and the Shepard diagram (cf. Borg & Groenen, 1997; Hair, Anderson, Tatham & Black, 1998).

In a scree plot, the amount of stress is plotted against the number of dimensions. Since stress decreases monotonically with increasing dimensionality, one is looking for the lowest number of dimensions with acceptable stress. An “elbow” in the scree plot indicates, that more dimensions would yield only a minor improvement in terms

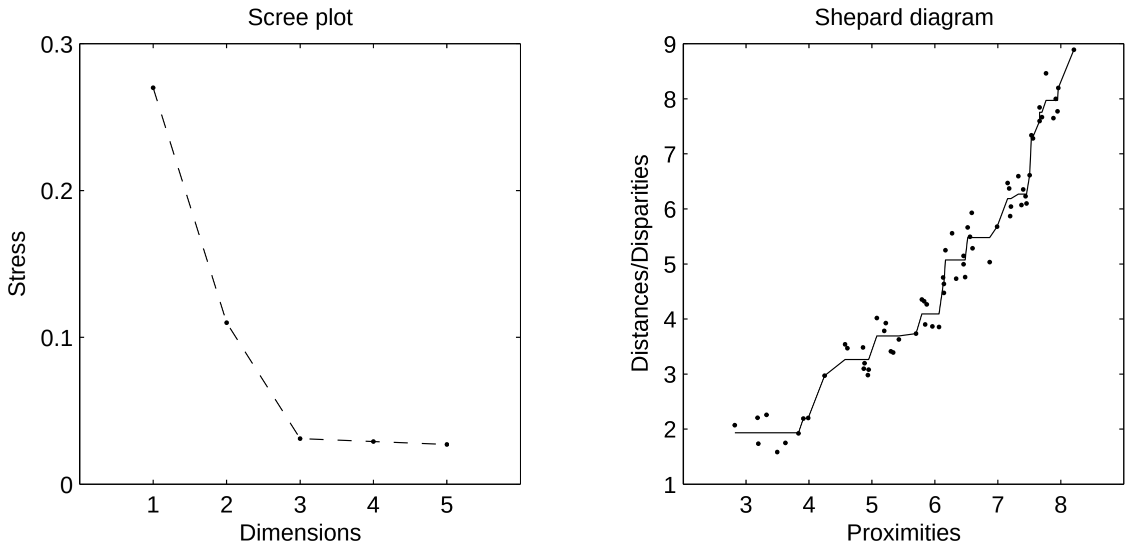

Figure 3: Left panel: Scree plot displaying an elbow at three dimensions. Right panel: Shepard diagram with the optimally scaled proximities.

of stress. Thus, the best fitting MDS model has as many dimensions as the number of dimensions at the elbow in the scree plot. The Shepard diagram displays the relationship between the proximities and the distances of the point configuration. Less spread in this diagram implies a good fit. In nonmetric MDS, the ideal location for the points in a Shepard diagram is a monotonically increasing line describing the so-called disparities, the optimally scaled proximities. In an MDS solution that fits well the points in the scree plot are close to this monotonically increasing line.

Figure 3 shows a paradigmatic scree plot and a Shepard diagram. The elbow in the scree plot suggests a three-dimensional MDS space, while the little amount of spread in the Shepard diagram indicates a rather good fit of the solution.

Basics of a nonmetric MDS algorithm

The core of a nonmetric MDS algorithm is a twofold optimization process. First the optimal monotonic transformation of the proximities has to be found. Secondly, the points of a configuration have to be optimally arranged, so that their distances match the scaled proximities as closely as possible. The basic steps in a nonmetric MDS algorithm are:

1. Find a random configuration of points, e. g. by sampling from a normal distribution.

2. Calculate the distances $\mathbf{d}$ between the points.

3. Find the optimal monotonic transformation of the proximities, in order to obtain optimally scaled data $f(\mathbf{p})$.

4. Minimize the stress between the optimally scaled data and the distances by finding a new configuration of points.

5. Compare the stress to some criterion. If the stress is small enough then exit the algorithm else return to 2.

3 Sound quality evaluation using MDS

The following section gives an example of an application of different types of MDS analyses in sound quality research. The data were collected at the Sound Quality Research Unit (SQRU), Aalborg University, in the summer of 2002 by Christian Schmid. One goal of the study was to reveal and identify the dimensions that subjects use in evaluating environmental sounds. A total number of 77 subjects participated in the experiment. They were presented with all 66 pairs of 12 environmental sounds via headphones. The subjects' task was to rate the dissimilarity of each two sounds on a scale from 1 (very similar) to 9 (very dissimilar). The resulting proximity matrices allow for both an individual and an aggregate analysis.

3.1 Individual analysis

For the individual analysis a nonmetric MDS model was chosen, since it is doubtful whether there is more than ordinal information in the data. Consider subject 37 as an example. Table 3 displays the proximity matrix, including the dissimilarity ratings.

With 12 stimuli it is possible to look for a spatial representation in at most three dimensions (cf. section 4). The scree plot in Figure 4 can help with finding the

sound	no.	1	2	3	4	5	6	7	8	9	10	11	12
circular saw	1	0	1	4	5	3	4	2	8	8	5	7	1
dentist's drill	2	1	0	3	8	2	6	2	7	8	8	7	2
fan	3	4	3	0	7	2	3	3	7	8	6	4	3
hooves	4	5	8	7	0	8	9	3	8	2	2	9	5
howling wind	5	3	2	2	8	0	6	4	8	9	6	4	2
ship's horn	6	4	6	3	9	6	0	1	3	9	8	5	3
stadium	7	2	2	3	3	4	1	0	4	3	6	7	1
stone in well	8	8	7	7	8	8	3	4	0	9	6	9	5
typewriter	9	8	8	8	2	9	9	3	9	0	4	9	9
tire on gravel	10	5	8	6	2	6	8	6	6	4	0	4	3
wasp	11	7	7	4	9	4	5	7	9	9	4	0	6
waterfall	12	1	2	3	5	2	3	1	5	9	3	6	0

Table 3: Individual proximity matrix of 12 environmental sounds for subject 37.

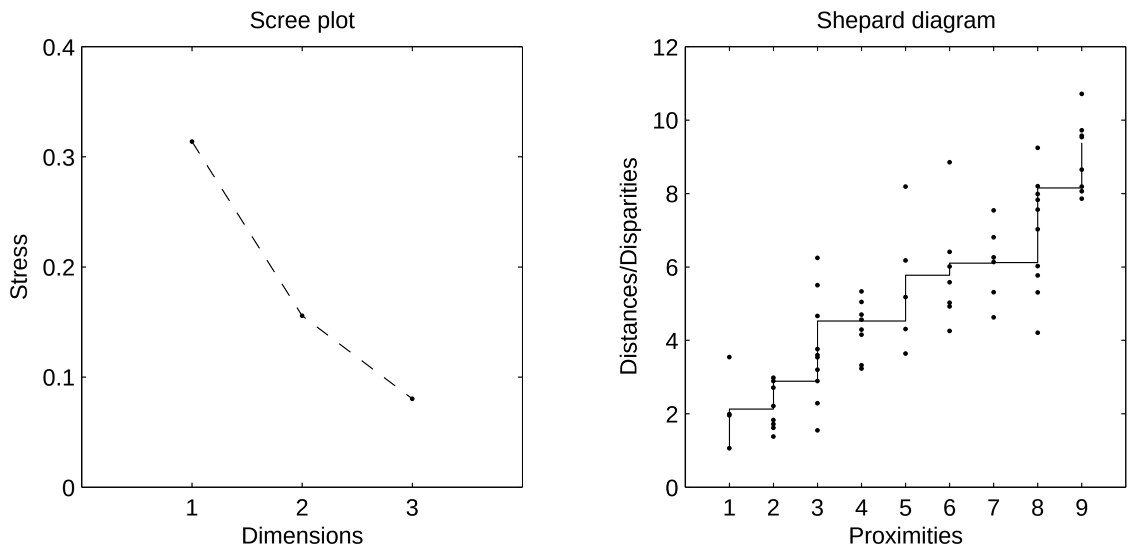

Figure 4: Scree plot and Shepard diagram for the two-dimensional MDS solution (subject 37).

appropriate number of dimensions. Obviously, a one-dimensional representation is not adequate (stress > .30). The scree plot does not display a clear elbow, but the largest improvement in terms of stress occurs when changing from one to two dimensions. Therefore, a two-dimensional solution was chosen. The stress for the two-dimensional solution is 0.156. The Shepard diagram displays the goodness of fit of the two-dimensional solution. The points of a perfectly fitting solution would lie on the monotonically increasing line. The spread in the Shepard diagram indicates some deviation from a perfect fit; it was, however, considered small enough to carry out further analyses.

In Figure 5 the two-dimensional graphical representation of the proximities of subject 37 is shown. When interpreting such an MDS map one strategy is to look for groups of objects. The three objects ‘typewriter’, ‘tire on gravel’, and ‘howling wind’ for example seem to form a group, since they are closer to each other than to any other sound. Another approach is to take two very distant objects and try to find an interpretation for the dimensions. The sounds ‘stone in well’ and ‘wasp’ seem to be very different on dimension two, but rather similar on dimension one. The substantial interpretation of these dimensions, however, is not obvious.

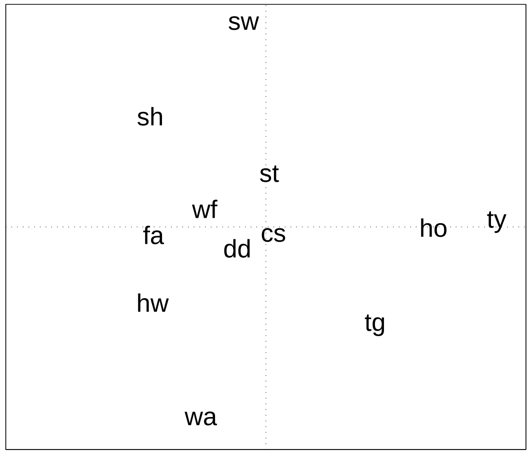

Figure 5: Individual MDS representation (subject 37) of twelve environmental sounds: circular saw (cs), dentist's drill (dd), fan (fa), hooves (ho), howling wind (hw), ship's horn (sh), stadium (st), stone in well (sw), typewriter (ty), tire on gravel (tg), wasp (wa), waterfall (wf).

3.2 Aggregate analysis

In order to perform an aggregate MDS analysis, the 77 single proximity matrices were combined by computing the average value for each cell. Again, a nonmetric MDS model with Euclidean distances was chosen to represent the data. The stress for a two-dimensional solution amounts to 0.106; the largest improvement in terms of stress occurs when changing from one to two dimensions. The graphical configuration is displayed in Figure 6.

If there are additional measurements of the stimuli available, it is possible to search for an empirical interpretation of the dimensions by correlating them with the external measure. In our case, from another experiment with the same sounds and largely the same subjects, an unpleasantness scale was derived. I. e. the unpleasantness value for each sound is known. Correlating the values on the first dimension (x-axis) with the unpleasantness scale using Spearman's rank correlation yields a statistically significant correlation of $\varrho = 0.69$. Roughly speaking, half of the variance along the x-axis in Figure 6 can be explained by the unpleasantness of the sounds. This finding emphasizes the importance of a psychological measure like unpleasantness for the perception of environmental sounds.

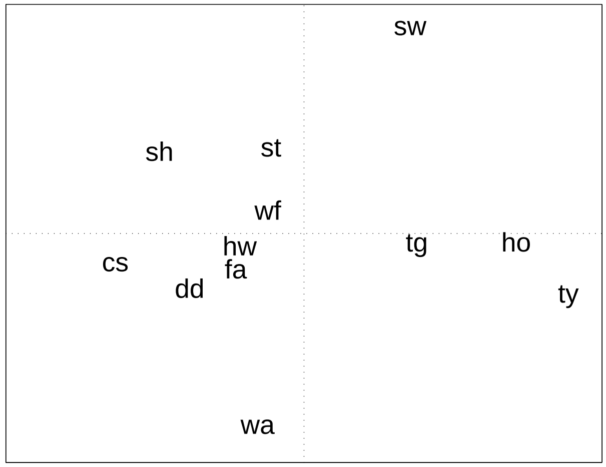

Figure 6: Aggregate MDS representation of twelve environmental sounds for 77 subjects (the labels of the sounds are the same as in Figure 5).

3.3 Individual difference scaling

Individual difference scaling (INDSCAL), or *weighted* MDS, was first introduced by Carrol & Chang (1970). Using this technique, it is possible to represent both the stimuli in a common MDS space, and the individual differences. In order to achieve this, the assumption is made that all subjects use the same dimensions when evaluating the objects, but that they might apply individual weights to these dimensions. By estimating the individual weights and plotting them (in the case of a low-dimensional solution) different groups of subjects can be detected. The input of an INDSCAL analysis are the individual proximity matrices of all subjects.

Figure 7 shows the outcome of the INDSCAL procedure as applied to the twelve environmental sounds and the sample of 77 participants presented in the previous section. The MDS map looks similar to the aggregate MDS representation depicted in Figure 6. The tight clustering of the subjects' weights in the right panel of Figure 7 reveals that the sample is rather homogeneous. Only the subjects 31, 36, 55, and 73 in the lower left corner of the figure may be regarded as a separate group; i. e. these participants put less weight on the two dimensions than the rest of the sample.

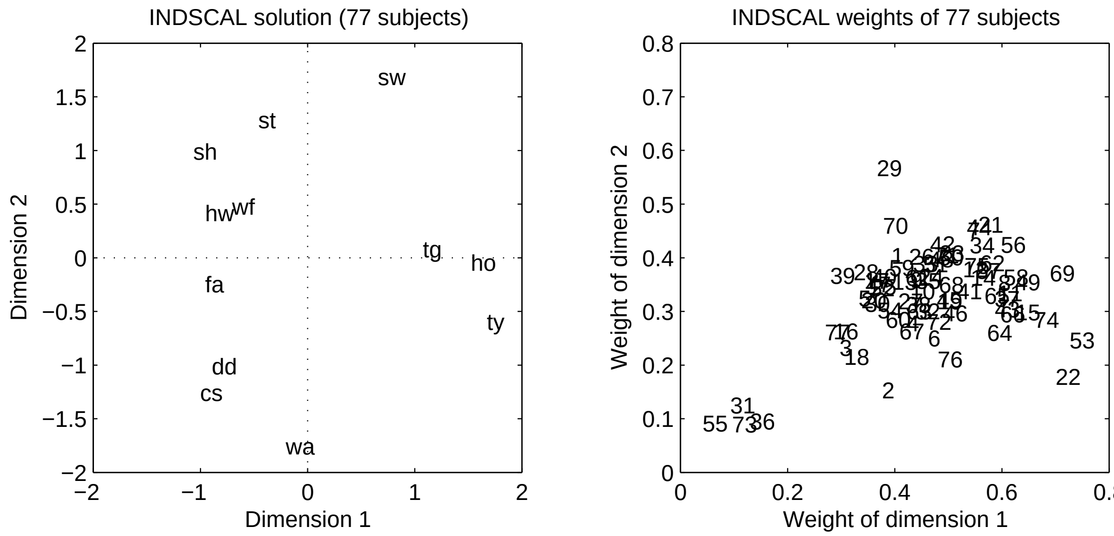

Figure 7: INDSCAL representation of twelve environmental sounds. The left panel displays the common two-dimensional MDS space, the right panel the weights of the two dimensions for each subject.

Conclusion

This section was set out to illustrate different MDS techniques as they might be employed in sound quality research. As a conclusion of the briefly described experiment it can be stated that (1) the subjects used largely the same dimensions when evaluating environmental sounds and (2) one of the evaluative dimensions was highly related to the perceived unpleasantness of the sounds.

4 Decisions to take before you start

MDS requires a certain amount of expertise on the part of the researcher. Unlike univariate statistical methods, the outcome of an MDS analysis is more dependent on the decisions that are taken beforehand. At the data-collection stage one should be aware that asking for similarity rather than for dissimilarity ratings might affect the results; i. e. a similarity judgment cannot simply be regarded as the "inverse" of a dissimilarity judgment. Further, a decision between direct and indirect methods, and symmetric versus asymmetric methods of data collection, respectively, has to be taken (cf. section 1).

The way the proximity matrix has been set up might already determine the choice of an appropriate MDS model. If the proximities are such that the actual numerical values are of little significance and the rank order is thought to be the only relevant information, then a nonmetric, rather than a metric, model should be chosen. Moreover, Euclidean distances are recommended whenever the most important goal of the analysis is to visualize the structure in the data; non-Euclidean distances will rather obscure the outcome from visual inspection, but they might be a valuable tool for investigating specific hypotheses about the subject's perceptual space. Finally, the type of stress measure chosen will affect the MDS representation.

Clearly, the number of dimensions of the MDS space will influence the solution most drastically. A-priori hypotheses might help to choose the appropriate number. If, for example, the question is whether or not it is possible to represent the objects by a unidimensional scale, one will be most interested in a one-dimensional MDS solution. The number of objects to be scaled gives further guidelines (Borg & Groenen, 1997): a k-dimensional representation requires at least $4k$ objects, i. e. a two-dimensional representation requires at least eight objects. A posteriori, the amount of stress and the number of *interpretable* dimensions will provide additional information on how many dimensions to choose.

Another important decision is the type of MDS analysis to be performed. Again the specific research question might determine the choice. An individual analysis represents the data of each subject most accurately. But often one is not interested in the individual differences, but in the perceptual space of an "average" subject. In this case, the aggregate analysis is more appropriate. INDSCAL provides both

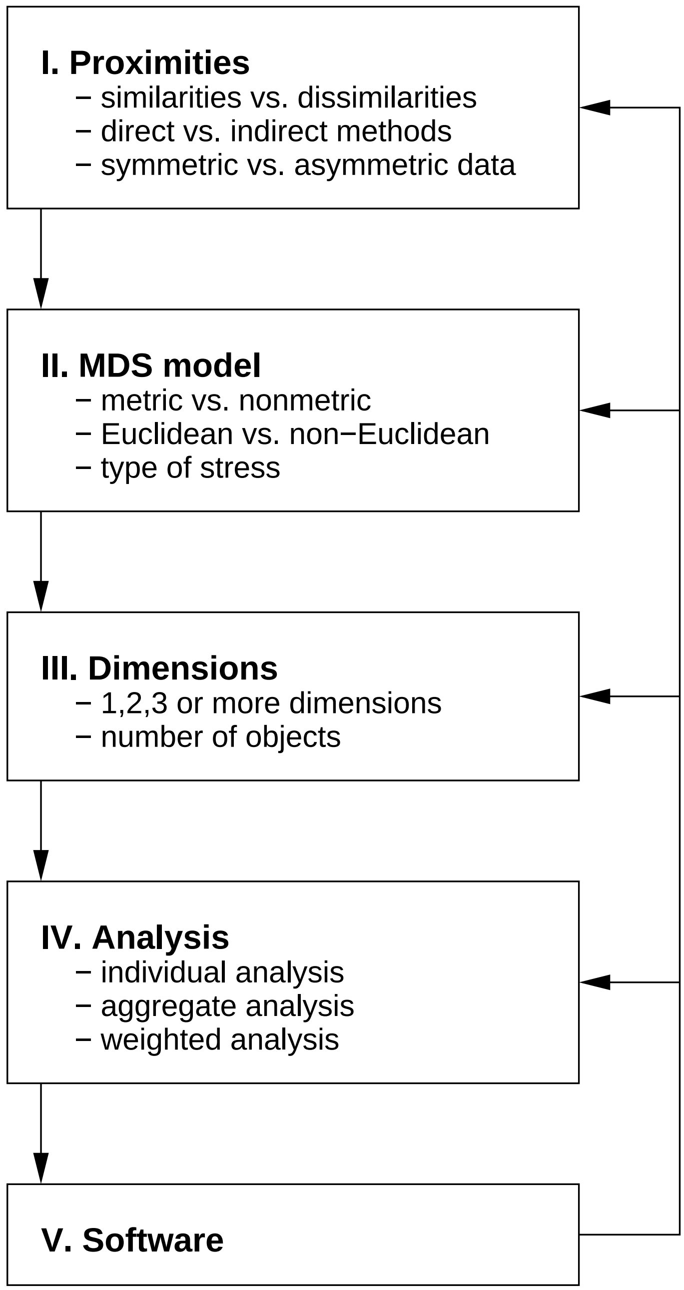

Figure 8: Diagram of important stages in an MDS analysis. At each stage a decision has to be taken by the researcher, which influences both the type of analysis and the outcome (see text for details).

a common MDS space and a representation of the individual differences. In that case, however, the assumption is made, that all subjects share a common perceptual space and differ only in their weights of the dimensions.

Finally, adequate MDS software has to be found that can do the analysis. All major statistical packages like SPSS, SAS, Statistica, and S-Plus, can perform MDS analyses. Aside from that stand-alone software exists, specially designed for MDS. One of the currently most used is ALSCAL by Forrest Young. It is freely available at `http://forrest.psych.unc.edu/research/alscal.html`.
However, with the choice of the software, some of the decisions at earlier stages might be predetermined, e. g. the type of stress measure or the type of analysis. When buying or using MDS software, one should re-assure that it can perform not only classical, but also nonmetric MDS. This holds for any of the above-mentioned software. Figure 8 shows the stages before an MDS analysis and some of the options the researcher has to choose among.

5 MDS literature

Multidimensional scaling now belongs to the standard techniques in statistics, even though it is rarely treated in introductory textbooks. Specific textbooks on multivariate data analysis cover topics in greater detail which are only briefly discussed in this paper. The following three books I found especially useful in gaining a basic understanding of MDS.

- Borg & Groenen (1997) provide a thorough introduction to multidimensional scaling. Features of MDS algorithms are outlined by many examples. Recommended to those who want to know more about what the computer programs actually do.

- An easy to read, application-oriented overview of MDS is given by Hair, Anderson, Tatham & Black (1998). This book covers many multivariate analysis techniques from the economist's point of view. Highly recommended to the reader with only basic statistical knowledge.

- Further information on the algorithm for individual difference scaling (INDSCAL) as well as on the ALSCAL algorithm can be found in Cox & Cox (1994). The theoretical results are illustrated by several examples.

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